

# lean-mode

emacs mode for Lean Theorem Prover

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# Features

- Show type/overload information at point
- On-the-fly syntax check
- Auto completion
- Jump to definition
- Set Lean options
- Eval Lean commands
- and More to come!

# Configuration

<https://github.com/leanprover/lean/blob/master/src/emacs/README.md>

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-- Author: Leonardo de Moura

import logic.axioms.hilbert logic.axioms.funext

open eq.ops nonempty inhabited

Diaconescu's theorem

Show that Excluded middle follows:

Hilbert's choice operator, function extensionality and Prop extensionality

context

hypothesis propext {a b : Prop} : (a → b) → (b → a) → a = b

parameter p : Prop

private definition u [reducible] := epsilon (λx, x = true v p)

private definition v [reducible] := epsilon (λx, x = false v p)

private lemma u\_def : u = true v p :=  
epsilon\_spec (exists.intro true (or.inl rfl))

private lemma v\_def : v = false v p :=  
epsilon\_spec (exists.intro false (or.inl rfl))

private lemma uv\_implies\_p : ¬(u = v) v p :=  
or.elim u\_def  
 (assume Hut : u = true, or.elim v\_def  
 (assume Hvf : v = false,  
 have Hne : ¬(u = v), from Hvf<sup>-1</sup> › Hut<sup>-1</sup> › true\_ne\_false,  
 or.inl Hne)  
 (assume Hp : p, or.inr Hp))  
 (assume Hp : p, or.inr Hp)

private lemma p\_implies\_uv : p → u = v :=  
assume Hp : p,  
 have Hpred : (λ x, x = true v p) = (λ x, x = false v p), from  
 funext (take x : Prop,  
 have Hl : (x = true v p) → (x = false v p), from  
 assume A, or.inr Hp,  
 have Hr : (x = false v p) → (x = true v p), from  
 assume B, or.inr Hp.

Show information at point  
(type, overloading, casting, etc)



```
private definition u [reducible] := epsilon (λx, x = true ∨ p)
private definition v [reducible] := epsilon (λx, x = false ∨ p)
```

```
private lemma u_def : u = true ∨ p :=
epsilon_spec (exists.intro true (or.inl rfl))
private lemma v_def : v = false ∨ p :=
epsilon_spec (exists.intro false (or.inl rfl))
```

Show type information of a sub-term in parens  
(put a cursor on a open-paren)

```
private lemma uv_implies_p : ¬(u = v) ∨ p :=
or.elim u_def
  (assume Hut : u = true, or.elim v_def
    (assume Hvf : v = false,
      have Hne : ¬(u = v), from Hvf⁻¹ ∙ Hut⁻¹ ∙ true_ne_false,
      or.inl Hne)
    (assume Hp : p, or.inr Hp))
  (assume Hp : p, or.inr Hp)
```

```
private lemma p_implies_uv : p → u = v :=
assume Hp : p,
have Hpred : (λ x, x = true ∨ p) = (λ x, x = false ∨ p), from
  funext (take x : Prop,
    have Hl : (x = true ∨ p) → (x = false ∨ p), from
      assume A, or.inr Hp,
    have Hr : (x = false ∨ p) → (x = true ∨ p), from
      assume A, or.inr Hp,
    show (x = true ∨ p) = (x = false ∨ p), from
      propext Hl Hr),
show u = v, from
  Hpred ∙ (eq.refl (epsilon (λ x, x = true ∨ p)))
```

```
theorem em : p ∨ ¬p :=
have H : ¬(u = v) → ¬p, from mt p_implies_uv,
or.elim uv_implies_p
  (assume Hne : ¬(u = v), or.inr (H Hne))
  (assume Hp : p, or.inl Hp)
end
```



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import logic.axioms.hilbert logic.axioms.funext

open eq.ops nonempty inhabited

### On-the-fly syntax check

Diaconescu's theorem

Show that Excluded middle follows from

Hilbert's choice operator, function extensionality and Prop extensionality

context

hypothesis propext {a b : Prop} : (a → b) → (b → a) → a = b

parameter p : Prop

private definition u [reducible] := epsilon (λx, x = true v p)

private definition v [reducible] := epsilon (λx, x = false v p)

private lemma u\_def : u = true v p :=  
epsilon\_spec (exists.intro true (or.inl rfl))

private lemma v\_def : v = false v p :=  
epsilon\_spec (exists.intro false (or.inl rfl))

private lemma uv\_implies\_p : ¬(u = v) v p :=

!or.elim u  
 (assume H : u = true, or.elim v\_def  
 (assume Hf : v = false,  
 have Huv : ¬(u = v), from Hvf<sup>-1</sup> > Hut<sup>-1</sup> > true\_ne\_false,  
 or.inl Huv)  
 (assume Hp : p, or.inr Hp))  
 (assume Hp : p, or.inr Hp)

private lemma p\_implies\_uv : p → u = v :=

assume Hp : p,  
 have Hpred : (λ x, x = true v p) = (λ x, x = false v p), from  
 funext (take x : Prop,  
 have Hl : (x = true v p) → (x = false v p), from  
 assume A, or.inr A,  
 have Hr : (x = false v p) → (x = true v p), from  
 assume A, or.inl A, Hp,

```
private definition v [reducible] := epsilon (λx, x = false ∨ p)
```

```
private lemma u_def : u = true ∨ p :=
epsilon_spec (exists.intro true (or.inl rfl))
```

```
private lemma v_def : v = false ∨ p :=
epsilon_spec (exists.intro false (or.inl rfl))
```

```
private lemma uv_implies_p : ¬(u = v) ∨ p :=
or.elim u
```

```
(assume Hut : u = true, or.elim v_def
  (assume Hvf : v = false,
    have Hne : ¬(u = v), from Hvf-1 ∙ Hut-1 ∙ true_ne_false,
    or.inl Hne)
  (assume Hp : p, or.inr Hp))
(assume Hp : p, or.inr Hp)
```

```
private lemma p_implies_uv : p → u = v :=
assume Hp : p,
```

On-the-fly syntax check  
C-c ! 1 : show list of errors

U:--- diaconescu.lean 33% (26,9) Git (Lean Hi ELDoc company MMM FlyC:2/0 GitGutter Projectile[lean] MRev guru Fill) 05

Line	Col	Level	ID	Message (Checker)
26	9	error		unknown identifier 'u'... (lean-checker)
49	11	error		unknown identifier 'uv_implies_p'... (lean-checker)



```
or.inl Hne)
  (assume Hp : p, or.inr Hp))
(assume Hp : p, or.inr Hp)
```

```
private lemma p_implies_uv : p → u = v :=
```

```
assume Hp : p,
have Hpred : (λ x, x = true ∨ p) = (λ x, x = false ∨ p), from
  funext (take x : Prop,
    have Hl : (x = true ∨ p) → (x = false ∨ p), from
      assume A, or.inr Hp,
    have Hr : (x = false ∨ p) → (x = true ∨ p), from
      assume A, or.inr Hp,
    show (x = true ∨ p) = (x = false ∨ p), from
      propext Hl Hr),
show u = v, from
  Hpred ▸ (eq.refl (epsilon (λ x, x = true ∨ p)))
```

Auto-completion with type  
tab

```
theorem em : p ∨ ¬p :=
have H : ¬(u = v) → ¬p, from mt p_implies_uv,
or.elim uv_implies_p
! (assume Hne : ¬(u = v), or.in (H Hne))
  (assume Hp : p, or.inl Hp)
end
```





```
or.inl Hne)
  (assume Hp : p, or.inr Hp))
(assume Hp : p, or.inr Hp)
```

```
private lemma p_implies_uv : p → u = v :=
```

```
assume Hp : p,
have Hpred : (λ x, x = true ∨ p) = (λ x, x = false ∨ p), from
  funext (take x : Prop,
    have Hl : (x = true ∨ p) → (x = false ∨ p), from
      assume A, or.inr Hp,
    have Hr : (x = false ∨ p) → (x = true ∨ p), from
      assume A, or.inr Hp,
    show (x = true ∨ p) = (x = false ∨ p), from
      propext Hl Hr),
show u = v, from
  Hpred ▸ (eq.refl (epsilon (λ x, x = true ∨ p)))
```

Auto-completion with type  
tab

```
theorem em : p ∨ ¬p :=
have H : ¬(u = v) → ¬p, from mt p_implies_uv,
or.elim uv_implies_p
```

```
! (assume Hne : ¬(u = v), or.inl (H Hne))
```

```
or.inr : b → a ∨ b
```

```
end or.inl : a → a ∨ b
```

```
or.intro_left : Π (b : Prop), a → a ∨ b
or.intro_right : ∀ (a : Prop) {b : Prop}, b → a ∨ b
or.induction_on : a ∨ b → (a → C) → (b → C) → C
bool.induction_on : Π (n : bool), C bool.ff → C bool.tt → C n
tactic.expr.induction_on : Π (n : tactic.expr), C tactic.expr.builtin → C n
prod.rprod.intro : Ra a1 a2 → Rb b1 b2 → prod.rprod Ra Rb (prod.mk a1 b1) (prod.mk a2 b2)
char.induction_on : Π (n : char), (Π (a a a a a a a a : bool), C (char.mk a a a a a a a a)) → C n
not.intro : (a → false) → ¬ a
option.induction_on : Π (n : option A), C option.none → (Π (a : A), C (option.some a)) → C n
prod.induction_on : Π (n : prod A B), (Π (pr1 : A) (pr2 : B), C (prod.mk pr1 pr2)) → C n
prod.rprod.induction_on : prod.rprod Ra Rb a a → (Π {a1 : A} {b1 : B} {a2 : A} {b2 : B}, Ra a1 a2 →...
```

```
have Hne : ¬(u = v), from Hvf-1 › Hut-1 › true_ne_false,
or.inl Hne)
(assume Hp : p, or.inr Hp))
(assume Hp : p, or.inr Hp)
```

```
private lemma p_implies_uv : p → u = v :=
assume Hp : p,
have Hpred : (λ x, x = true ∨ p) = (λ x, x = false ∨ p), from
funext (take x : Prop,
have Hl : (x = true ∨ p) → (x = false ∨ p), from
assume A, or.inr Hp,
have Hr : (x = false ∨ p) → (x = true ∨ p), from
assume A, or.inr Hp,
show (x = true ∨ p) = (x = false ∨ p), from
propext Hl Hr),
show u = v, from
Hpred › (eq.refl (epsilon (λ x, x = true ∨ p)))
```

Jump to definition  
M-

```
theorem em : p ∨ ¬p :=
have H : ¬(u = v) → ¬p, from mt p_implies_uv,
or.elim uv_implies_p
(assume Hne : ¬(u = v), or.inr (H Hne))
(assume Hp : p, or.inl Hp)
end
```



```

have Hne : ¬(u = v), from Hvf-1 › Hut-1 › true_ne_false,
or.inl Hne)
(assume Hp : p, or.inr Hp))
(assume Hp : p, or.inr Hp)

```

```

private lemma p_implies_uv : p → u = v :=
assume Hp : p,
have Hpred : (λ x, x = true ∨ p) = (λ x, x = false ∨ p), from
funext (take x : Prop,
have Hl : (x = true ∨ p) → (x = false ∨ p), from
assume A, or.inr Hp,
have Hr : (x = false ∨ p) → (x = true ∨ p), from
assume A, or.inr Hp,
show (x = true ∨ p) = (x = false ∨ p), from
propext Hl Hr),
show u = v, from
Hpred › (eq.refl (epsilon (λ x, x = true ∨ p)))

```

Jump to definition  
M-

```

theorem em : p ∨ ¬p :=
have H : ¬(u = v) → ¬p, from mt p_implies_uv,
or.elim uv_implies_p
(assume Hne : ¬(u = v), or.inr (H Hne))
(assume Hp : p, or.inl Hp)
end

```

Can take a few seconds  
for the first time.

```
calc_trans heq.of_heq_of_eq
calc_trans heq.of_eq_of_heq
calc_symm heq.symm
```

*/- and -/*

```
notation a ∧ b := and a b
notation a ∧ b := and a b
variables {a b c d : Prop}
```

Jump to definition  
M-.

```
theorem and.elim (H1 : a ∧ b) (H2 : a → b → c) : c :=
and.rec H2 H1
```

*/- or -/*

```
notation a ∨ b := or a b
notation a ∨ b := or a b
```

M-\* will pop you back!

```
namespace or
| theorem elim (H1 : a ∨ b) (H2 : a → c) (H3 : b → c) : c :=
  rec H2 H3 H1
end or
```



*/- iff -/*

```
definition iff (a b : Prop) := (a → b) ∧ (b → a)
```

```
notation a <-> b := iff a b
notation a ↔ b := iff a b
```

namespace iff

```
definition intro (H1 : a → b) (H2 : b → a) : a ↔ b :=
and.intro H1 H2
```

```
definition elim (H1 : (a → b) → (b → a) → c) (H2 : a ↔ b) : c :=
and.rec H1 H2
```

```
definition elim_left (H : a ↔ b) : a → b :=
elim (assume H1 H2, H1) H
```

```

or.elim u_def
  (assume Hut : u = true, or.elim v_def
    (assume Hvf : v = false,
      have Hne : ¬(u = v), from Hvf-1 > Hut-1 > true_ne_false,
      or.inl Hne)
    (assume Hp : p, or.inr Hp))
  (assume Hp : p, or.inr Hp)

```

Set lean options  
C-c C-o

```

private lemma p_implies_uv : p → u = v :=
assume Hp : p,
have Hpred : (λ x, x = true ∨ p) = (λ x, x = false ∨ p), from

```

```

funext (take x : Prop,
  have Hl : (x = true ∨ p) → (x = false ∨ p), from
    assume A, or.inr Hp,
  have Hr : (x = false ∨ p) → (x = true ∨ p), from
    assume A, or.inr Hp,
  show (x = true ∨ p) = (x = false ∨ p), from
    propext Hl Hr),
show u = v, from
  Hpred > (eq.refl (epsilon (λ x, x = true ∨ p)))

```

```

□
theorem em : p ∨ ¬p :=
have H : ¬(u = v) → ¬p, from mt p_implies_uv,
or.elim uv_implies_p
  (assume Hne : ¬(u = v), or.inr (H Hne))
  (assume Hp : p, or.inl Hp)

```

end



```

or.elim u_def
  (assume Hut : u = true, or.elim v_def
    (assume Hvf : v = false,
      have Hne : ¬(u = v), from Hvf-1 > Hut-1 > true_ne_false,
      or.inl Hne)
    (assume Hp : p, or.inr Hp))
  (assume Hp : p, or.inr Hp)

```

Set lean options  
C-c C-o

```

private lemma p_implies_uv : p → u = v :=
assume Hp : p,
have Hpred : (λ x, x = true ∨ p) = (λ x, x = false ∨ p), from

```

```

funext (take x : Prop,
  have Hl : (x = true ∨ p) → (x = false ∨ p), from
    assume A, or.inr Hp,
  have Hr : (x = false ∨ p) → (x = true ∨ p), from
    assume A, or.inr Hp,
  show (x = true ∨ p) = (x = false ∨ p), from
    propext Hl Hr),

```

```

show u = v, from
  Hpred > (eq.refl (epsilon (λ x, x = true ∨ p)))

```

```

□
theorem em : p ∨ ¬p :=
have H : ¬(u = v) → ¬p, from mt p_implies_uv,
or.elim uv_implies_p
  (assume Hne : ¬(u = v), or.inr (H Hne))
  (assume Hp : p, or.inl Hp)

```

end



```

or.elim u_def
  (assume Hut : u = true, or.elim v_def
    (assume Hvf : v = false,
      have Hne : ¬(u = v), from Hvf-1 > Hut-1 > true_ne_false,
      or.inl Hne)
    (assume Hp : p, or.inr Hp))
  (assume Hp : p, or.inr Hp)

```

Evaluate lean commands  
C-c C-e

```

private lemma p_implies_uv : p → u = v :=
assume Hp : p,
have Hpred : (λ x, x = true v p) = (λ x, x = false v p), from
  funext (take x : Prop,
    have Hl : (x = true v p) → (x = false v p), from
      assume A, or.inr Hp,
    have Hr : (x = false v p) → (x = true v p), from
      assume A, or.inr Hp,
    show (x = true v p) = (x = false v p), from
      propext Hl Hr),
show u = v, from
  Hpred > (eq.refl (epsilon (λ x, x = true v p)))
□
theorem em : p ∨ ¬p :=
have H : ¬(u = v) → ¬p, from mt p_implies_uv,
or.elim uv_implies_p
  (assume Hne : ¬(u = v), or.inr (H Hne))
  (assume Hp : p, or.inl Hp)
end

```



```
or.elim u_def
  (assume Hut : u = true, or.elim v_def
    (assume Hvf : v = false,
      have Hne : ¬(u = v), from Hvf-1 > Hut-1 > true_ne_false,
      or.inl Hne)
    (assume Hp : p, or.inr Hp))
  (assume Hp : p, or.inr Hp)
```

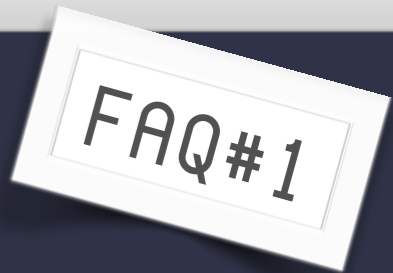
Evaluate lean commands  
C-c C-e

```
private lemma p_implies_uv : p → u = v :=
  assume Hp : p,
  have Hpred : (λ x, x = true v p) = (λ x, x = false v p), from
  funext (take x : Prop,
    have Hl : (x = true v p) → (x = false v p), from
      assume A, or.inr Hp,
    have Hr : (x = false v p) → (x = true v p), from
      assume A, or.inr Hp,
    show (x = true v p) = (x = false v p), from
      propext Hl Hr),
  show u = v, from
  Hpred > (eq.refl (epsilon (λ x, x = true v p)))
  ─
theorem em : p ∨ ¬p :=
  have H : ¬(u = v) → ¬p, from mt p_implies_uv,
  or.elim uv_implies_p
    (assume Hne : ¬(u = v), or.inr (H Hne))
    (assume Hp : p, or.inl Hp)
end
```





# FAQS



```

assume Hp : p,
have Hpred : (λ x, x = true v p) = (λ x, x = false v p), from
funext (take x : Prop,
  have H1 : (x = true v p) = (x = false v p), from
  have H2 : (x = false v p) = (x = true v p), from
  show (x = true v p) = (x = false v p), from
  propext (H1 H2)),
show u = v, from
Hpred ▯ (eq.refl (epsilon (λ x, x = true v p)))

```

Q: How can I type this symbol '▯' ?

```

theorem em : p v ¬p :=
have H : ¬(u = v) → ¬p, from mt p_implies_uv,
or.elim uv_implies_p
  (assume Hne : ¬(u = v), or.inr (H Hne))
  (assume Hp : p, or.inl Hp)
end

```

FAQ#1

```

assume Hp : p,
have Hpred : (λ x, x = true ∨ p) = (λ x, x = false ∨ p), from
  funext (take x : Prop,
    have H1 : (x = true ∨ p) = (x = false ∨ p), from
      propext (Hr),
    show u = v, from
      Hpred (eq.refl (epsilon (λ x, x = true ∨ p)))
  )

```

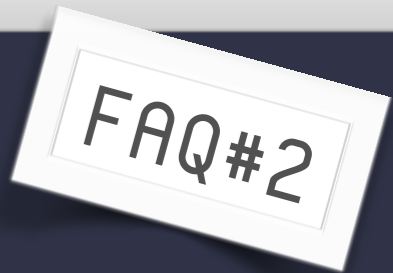
A: Press 'C-c C-k'!

```

theorem em : p ∨ ¬p :=
have H : ¬(u = v) → ¬p, from mt p_implies_uv,
or.elim uv_implies_p
  (assume Hne : ¬(u = v), or.inr (H Hne))
  (assume Hp : p, or.inl Hp)
end

```





```

assume Hp : p,
have Hpred : (λ x, x = true v p) = (λ x, x = false v p), from
  funext (take x : Prop,
    have Hl : (x = true v p) → (x = false v p), from
      assume A, or.inr Hp,
    have Hr : (x = false v p) → (x = true v p), from
      assume A, or.inr Hp,
    show (x = true v p) = (x = false v p), from
      propext Hl Hr),
show u = v, from
  Hpred ▯ (eq.refl (epsilon (λ x, x = true v p)))

```

```

theorem em : p v ¬p :=
have H : ¬(u = v) → ¬p, from mt p_implies_uv,
or.elim uv_implies_p
  (assume Hne : ¬(u = v), or.inr (H Hne))
  (assume Hp : p, or.inl Hp)
end

```

Q: It seems that nothing is working!  
What should I do?

FAQ#2

```

assume Hp : p,
have Hpred : (λ x, x = true v p) = (λ x, x = false v p), from
  funext (take x : Prop,
    have Hl : (x = true v p) → (x = false v p), from
      assume A, or.inr Hp,
    have Hr : (x = false v p) → (x = true v p), from
      assume A, or.inr Hp,
    show (x = true v p) = (x = false v p), from
      propext Hl Hr),
show u = v, from
  Hpred ▯ (eq.refl (epsilon (λ x, x = true v p)))

```

```

theorem em : p v ¬p :=
have H : ¬(u = v) → ¬p, from mt p_implies_uv,
or.elim uv_implies_p
  (assume Hne : ¬(u = v), or.inr (H Hne))
  (assume Hp : p, or.inl Hp)
end

```

A: Keep calm and run  
 "M-x lean-server-restart-process"  
 then please file a bug report!  
 (with reproducible steps)

# Bug Reports, Feature Requests

<https://github.com/leanprover/lean/issues/new>

Contributions are Welcome!