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Towards Lean 4: An Optimized Object Model for an Interactive Theorem Prover

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LEAN THEOREM PROVER

The Lean theorem prover

- dependently-typed proof assistant
- small trusted kernel
- also a purely functional, eager programming language

```
inductive list (α : Type u)
| nil : list
| cons : α → list → list
```

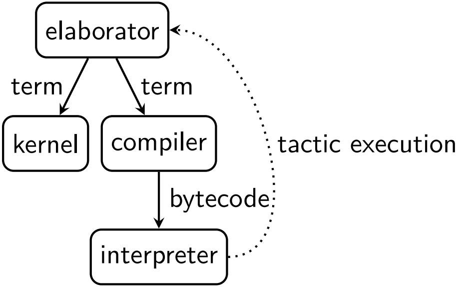
```
def map (f : α → β) : list α → list β
| []          := []
| (x :: xs') := f x :: map xs'
```

https://leanprover.github.io

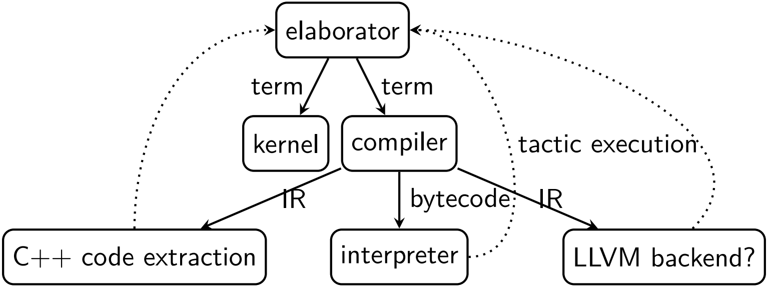
A brief history of Lean

- Lean 0.1 (2014)
- Lean 2 (2015)
  - first official release
  - fixed tactic language
- Lean 3 (2017)
  - make Lean a meta-programming language: build tactics in Lean
  - backed by a bytecode interpreter
- Lean 4 (201X)
  - make Lean a general-purpose language: native back end, FFI, ...
  - reimplement Lean in Lean

Lean 3 backend



Lean 4 backend



Lean 3 object model

Uniform model: every value is a tagged pointer representing one of

- a 31-bit number
- a reference to a ref-counted VM object
  - a constructor value
  - a closure
  - an arbitrary-precision integer
  - any C++ object derived from `vm_external`

Lean 3 constructor object

4 bytes	reference counter
1 byte	object kind
4 bytes	constructor index
4 bytes	#fields
4/8 bytes	field #0
...	...

Lessons from Lean 3's model

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- Originally only intended for single-threaded code
  $\Longrightarrow$ no need for atomic RC!

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  $\Longrightarrow$ no need for atomic RC!
- Eventually needed to move objects between threads
  $\Longrightarrow$ fall back to deep-copying...
- Every object is a C++ smart pointer
  $\Longrightarrow$ simple to use, but no way to optimize RC ops
- Core types like `name` and `expr` are not VM objects
  $\Longrightarrow$ need to be wrapped in `vm_external` for every operation

Lean 4 object model

Non-uniform model: in the lowest IR, each value has one of the types

- `int8/uint8/.../uint64`: unboxed primitive value
- `_obj`: tagged pointer to a VM object
  - a constructor, closure, or bigint
  - an array of boxed or unboxed values
  - a thunk

Lean 4 constructor object

1 byte	object kind
1 byte	memory kind
2 bytes	constructor index
2 bytes	#boxed fields
2 bytes	#unboxed bytes
4/8 bytes	boxed field #0
...	...
X bytes	unboxed field #0
...	...

All boxed fields come first $\Longrightarrow$ `free` can still be implemented uniformly

Memory kind

- single-threaded: non-atomic RC
  - the default for heap allocations
- multi-threaded: atomic RC
  - threading primitives *upgrade* object graphs crossing threads to this kind
  - everything is immutable $\Longrightarrow$ ST object never reachable from MT object
- stack: no RC
- region: no RC

The case for ref counting

- writing a good GC is really hard

  "The biggest challenge is implementing the garbage collector."

  – Multicore OCaml website[1]

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- easier to use from other languages
- everything is immutable $\Longrightarrow$ no cycles!
- explicit ref count $\Longrightarrow$ can do destructive updates on $RC = 1$
  - like linear types, but checked dynamically
    - dependent types are hard enough
    - more precise (but also less predictable)

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Dynamic linearity

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```
[compiler.llnf]
λ (f xs : _obj),
  list.cases_on xs
    (let _x_1 : _obj := _dec f
     in _cnstr.0)
    (let _x_1 : _obj := _proj.0 xs,
         _x_2 : _obj := _inc _x_1,
         _x_3 : _obj := _proj.1 xs,
         _x_4 : _obj := _inc _x_3,
         _x_5 : _obj := _reset.2 xs,
         _x_6 : _obj := _apply f _x_2,
         _x_7 : _obj := list.map f _x_4
     in _reuse.1 _x_5 _x_6 _x_7)
```

_reset / _reuse check for linearity at runtime

⟹ unique prefix of a list will be reused even if remainder is shared!

Dynamic linearity

Benchmarks of direct C++ implementations of

```
list.map (+1) (list.range 4000)
```

optimizations	run time of `map`
no reuse	214.3 μs
_reset / _reuse	27.7 μs
optimized reuse	12.3 μs
known unique	10.7 μs

Borrowing

```
def length : @borrowed (list α) → nat
| []         := 0
| (x :: xs') := length xs' + 1
```

```
[compiler.llnf]
λ (xs : _obj),
  list.cases_on xs
    0
    (let _x_1 : _obj := _proj.1 xs,
         _x_2 : _obj := length _x_1,
     in nat.add _x_2 1)
```

The `@borrowed` attribute

- delays/avoids RC operations:
  - no inc/dec when passing an argument to a borrow parameter
  - inc when returning/passing a borrowed value to a non-borrow parameter
- but prevents linear updates

Regions: minimizing startup time

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What if we could just `mmap` (multiple!) regions of objects into memory?

- lazy loading and prefetching provided by the OS
  - proofs aren't needed usually
- everything immutable
  $\Longrightarrow$ pages can even be shared by multiple Lean processes
  - careful: must not touch RC

Regions: minimizing startup time

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We're investigating two approaches:

Simple approach: use relative pointers in region objects

- introduces branch for retrieving unboxed field

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Advanced approach: try to `mmap` each region to its original address

- on collision: fall back to eager loading and pointer patching
- probability of a single collision between 100 dependencies of size 10 MB in 48-bit address space is ∼0.018%

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In either approach, *writing* objects to disk does need some transformations

Regions: minimizing startup time

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Unboxed fields now make it feasible to reimplement core types as Lean objects!

```
expr mk_const(name const & n, levels const & ls) {
    expr r(mk_cnstr(static_cast<unsigned>(expr_kind::Const), n, ls, expr_scalar_size(expr_kind::Const)));
    set_scalar<expr_kind::Const>(r, hash(n.hash(), hash(ls)), false, has_mvar(ls), false, has_param(ls));
    return r;
}
```

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    return r;
}
```

Unboxed metadata is at the end of the object
$\Longrightarrow$ can be hidden in the Lean definition

```
inductive expr
| const : name → list level → expr
| ...
```

Implementation status

- object model runtime in C++
- core types ported to model
- optimizing compiler from Core Lean to LLNF
  - inlining, specialization, simplification
  - using join-point representation
- compiler from LLNF to old bytecode format
- model used by backends and built-ins
- writing and loading regions
- multi-threading
- borrowing

Conclusion

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- utilizing properties of an eager, purely functional language
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Thank you!